2021/2022 FIRST

QUESTION ONE

$\dot{x} = Ax + Bu \qquad y = Cx + Du$

(a) $\dot{x}$ = Dynamic matrix

x = state vector

y = output vector

u = input vector

A = state matrix

B = input/control matrix

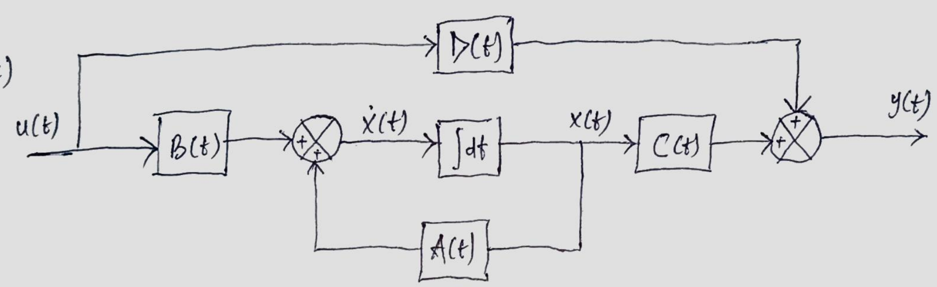
C = output matrix

D = transmission matrix

(b) First equation; $\dot{x} = Ax + Bu \rightarrow$ State equation

Second equation: $y = Cx + Du \rightarrow$ Output equation

(c)



$u = [V_a \ \ V_b]$

(d) $R i_2(t) + L_1 \frac{di_1}{dt} + v(t) = V_a(t)$

$L_2 \frac{di_2}{dt} + v(t) = V_b(t)$

$i_1(t) + i_2(t) = C \frac{dv(t)}{dt}$

$V_a(t), V_b(t)$ = inputs

$V_a(t) = V_b(t) = u$

$x_1(t) = i_1(t)$

$x_2(t) = i_2(t)$

$x_3(t) = v(t)$

$\frac{di_1}{dt} = \frac{dx_1}{dt}$

$\frac{di_2}{dt} = \frac{dx_2}{dt}$

$R x_2(t) + L_1 \dot{x}_1(t) + x_3(t) = V_a(t)$ —— I

$L_2 \dot{x}_2 + x_3(t) = V_b(t)$ —— II

$x_1(t) + x_2(t) = C \dot{x}_3(t)$ —— III

Express equation (I) in terms of $\dot{x}_1(t)$

" " (II) in terms of $\dot{x}_2(t)$

" " III " " " $\dot{x}_3(t)$

$$R x_2(t) + L_1 \dot{x}_1(t) + x_3(t) = V_a(t)$$

$$L_1 \dot{x}_1(t) = -R x_2(t) - x_3(t) + V_a(t)$$

$$\dot{x}_1(t) = \frac{-R}{L_1} x_2(t) - \frac{1}{L_1} x_3(t) + \frac{1}{L_1} V_a(t) \quad \text{—— iv}$$

$$L_2 \dot{x}_2(t) + x_3(t) = V_b(t)$$

$$L_2 \dot{x}_2(t) = -x_3(t) + V_b(t)$$

$$\dot{x}_2(t) = \frac{-1}{L_2} x_3(t) + \frac{1}{L_2} V_b(t) \quad \text{—— v}$$

$$x_1(t) + x_2(t) = C \dot{x}_3(t)$$

$$C \dot{x}_3(t) = x_1(t) + x_2(t)$$

$$\dot{x}_3(t) = \frac{1}{C} x_1(t) + \frac{1}{C} x_2(t) \quad \text{—— vi}$$

$$\dot{x} = Ax + Bu$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & -R/L_1 & -1/L_1 \\ 0 & 0 & -1/L_2 \\ 1/C & 1/C & 0 \end{bmatrix} + \begin{bmatrix} 1/L_1 \\ 1/L_2 \\ 0 \end{bmatrix} u$$

output is $x_3(t)$

$$y = x_3(t)$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

QUESTION TWO

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \quad X(0) = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$L^{-1}\{[sI-A]^{-1}[X(0)+Bu]\}$

Transition matrix $= \mathcal{L}^{-1}\{[sI-A]^{-1}\}$

$$X(t) = L^{-1}\{[sI-A]^{-1}[X(0) + Bu(s)]\}$$

$$sI - A = s\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 6 & 11 & s+6 \end{bmatrix} \qquad \begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

To get inverse, compute the cofactor and transpose it = adjoint

$$a_{11} = \begin{vmatrix} s & -1 \\ 11 & s+6 \end{vmatrix} = s(s+6) - (-1\times 11) = s^2+6s+11$$

$$a_{12} = -\begin{vmatrix} 0 & -1 \\ 6 & s+6 \end{vmatrix} = -[0 - (-1\times 6)] = -[6] = -6$$

$$a_{13} = \begin{vmatrix} 0 & s \\ 6 & 11 \end{vmatrix} = 0\times 11 - 6\times s = -6s$$

$$a_{21} = -\begin{vmatrix} -1 & 0 \\ 11 & s+6 \end{vmatrix} = -[-(s+6) - 0] = s+6$$

$$a_{22} = \begin{vmatrix} s & 0 \\ 6 & s+6 \end{vmatrix} = s(s+6) + 0 = s^2+6s$$

$$a_{23} = -\begin{vmatrix} s & -1 \\ 6 & 11 \end{vmatrix} = -[11s - (-6)] = -[11s+6] = -11s - 6$$

$$a_{31} = \begin{vmatrix} -1 & 0 \\ s & -1 \end{vmatrix} = (-1\times -1) - (0\times s) = 1$$

$$a_{32} = -\begin{vmatrix} s & 0 \\ 0 & -1 \end{vmatrix} = -[\underbrace{-s - 0}_{-s}] = s$$

$$a_{33} = \begin{vmatrix} s & -1 \\ 0 & s \end{vmatrix} = s^2$$

$$\text{Cofactor} \begin{bmatrix} s^2+6s+11 & -6 & -6s \\ s+6 & s^2+6s & -11s-6 \\ 1 & s & s^2 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{Adj(sI-A)}{det(sI-A)}$$

$$\text{Adjoint}(sI-A) = \begin{bmatrix} s^2+6s+11 & s+6 & 1 \\ -6 & s^2+6s & s \\ -6s & -11s-6 & s^2 \end{bmatrix}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ab - bc$$

$$\begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ |sI-A| = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 6 & 11 & s+6 \end{vmatrix} \end{array} = +s\begin{vmatrix} s & -1 \\ 11 & s+6 \end{vmatrix} - (-1)\begin{vmatrix} 0 & -1 \\ 6 & s+6 \end{vmatrix} + 0\begin{vmatrix} 0 & s \\ 6 & 11 \end{vmatrix}$$

$$= s\left(s(s+6) - (-1)(11)\right) + \left(0 - (-6)\right) + 0$$

$$= s\left(s^2+6s+11\right) + 6 \qquad s=-3 \qquad s=-2$$

$$= s^3+6s^2+11s+6 \qquad s=-1$$

$$= (s+1)(s+2)(s+3)$$

$$(sI-A)^{-1} = \frac{1}{(s+1)(s+2)(s+3)} \begin{bmatrix} s^2+6s+11 & s+6 & 1 \\ -6 & s^2+6s & s \\ -6s & -11s-6 & s^2 \end{bmatrix}$$

$3 \times 3 \quad 3 \times 1$

$$x(0) + Bu(s) = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}$$

$$[sI-A]^{-1}[x(0)+Bu] = \frac{1}{(s+1)(s+2)(s+3)} \begin{bmatrix} 4 \\ 4s \\ 4s^2 \end{bmatrix} = \begin{bmatrix} \frac{4}{(s+1)(s+2)(s+3)} \\ \frac{4s}{(s+1)(s+2)(s+3)} \\ \frac{4s^2}{(s+1)(s+2)(s+3)} \end{bmatrix}$$

$$\frac{4}{(s+1)(s+2)(s+3)} \equiv \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

multiply through by $(s+1)(s+2)(s+3)$

$$4 \equiv A(s+2)(s+3) + B(s+1)(s+3) + C(s+1)(s+2)$$

Solve for A set $s=-1$

$$4 = A(-1+2)(-1+3)$$
$$4 = A(1)(2)$$
$$4 = 2A$$
$$A = 2$$

Solve for B set $s=-2$

$$4 = B(-2+1)(-2+3)$$
$$4 = B(-1)(1)$$
$$4 = -B$$
$$B = -4$$

Solve for C, set $s=-3$

$$4 = C(-3+1)(-3+2)$$
$$4 = C(-2)(-1)$$
$$4 = C(2)$$
$$4 = 2C$$
$$C = 2$$

$$\boxed{\begin{aligned}\frac{4}{(s+1)(s+2)(s+3)} &\equiv \frac{2}{s+1} - \frac{4}{s+2} + \frac{2}{s+3}\\ \frac{4s}{(s+1)(s+2)(s+3)} &\equiv \frac{-2}{s+1} + \frac{8}{s+2} - \frac{6}{s+3}\\ \frac{4s^2}{(s+1)(s+2)(s+3)} &\equiv \frac{2}{s+1} - \frac{16}{s+2} + \frac{18}{s+3}\end{aligned}}$$

$$\frac{4s}{(s+1)(s+2)(s+3)} \equiv \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

Solve for A, set $s=-1$

$$4s = A(-1+2)(-1+3)$$
$$4(-1) = 2A$$
$$-4 = 2A$$
$$A = -2$$

Solve for B, set $s=-2$

$$4(-2) = B(-2+1)(-2+3)$$
$$-8 = B(-1)(1)$$
$$-8 = -B$$
$$B = 8$$

Solve for C, set $s=-3$

$$4(-3) = C(-3+1)(-3+2)$$
$$4(-3) = C(-2)(-1)$$
$$-12 = 2C$$
$$C = -6$$

$$\frac{4s^2}{(s+1)(s+2)(s+3)} \equiv \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

Solve for A, set $s=-1$

$$4s^2 = A(-1+2)(-1+3)$$
$$4(-1)^2 = A(1)(2)$$
$$4 = 2A$$
$$A = 2$$

Solve for B, set $s=-2$

$$4s^2 = B(-2+1)(-2+3)$$
$$4(-2)^2 = B(-1)(1)$$
$$16 = -B$$
$$B = -16$$

Solve for C, set $s=-3$

$$4s^2 = C(-3+1)(-3+2)$$
$$4(-3)^2 = C(-2)(-1)$$
$$36 = 2C$$
$$C = 18$$

$$[sI-A]^{-1}[x(0)+Bu] = \begin{bmatrix} \frac{2}{s+1} - \frac{4}{s+2} + \frac{2}{s+3} \\ \frac{-2}{s+1} + \frac{8}{s+2} - \frac{6}{s+3} \\ \frac{2}{s+1} - \frac{16}{s+2} + \frac{18}{s+3} \end{bmatrix}$$

NOTE

$$L^{-1}\left\{\frac{1}{x+1}\right\} = e^{-t}$$

$$L^{-1}\left\{\frac{1}{x-1}\right\} = e^{t}.$$

$$L^{-1}\left\{[sI-A]^{-1}[x(0)+Bu]\right\} = \begin{bmatrix} 2e^{-t} - 4e^{-2t} + 2e^{-3t} \\ -2e^{-t} + 8e^{-2t} - 6e^{-3t} \\ 2e^{-t} - 16e^{-2t} + 18e^{-3t} \end{bmatrix}$$

QUESTION THREE

(a) (i) 1- Full order state observer

2- Minimum order state observer

3- Reduced order state observer

x_1, x_2, x_3, x_4 & x_5

x_2 and x_3 are available for measurement

1- Full order state observer estimates all state variables whether they are available for measurement or not.

2- Minimum order state observer estimates only those variables that are not available for measurement.

3- Reduced order state observer estimates all those state variables that are not available for measurement and few remaining states variables available for measurement.

(ii) Controllability: A control system is set to be controllable if the initial state of the system are changed to some other desired state by a controlled input in a finite duration of time.

(iii) Observability: A control system is said to be observable if it is able to determine the initial state of the control system by observing the output in finite duration of time.

(b) $\dot{x} = Ax + Bu \qquad y = Cx \qquad$ where $A = \begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\mu_1 = -1.8 + 2.4j \qquad \mu_2 = -1.8 - 2.4j \qquad C = [0 \quad 1]$

SUBSTITUTION METHOD

Step 1: Check for observability

$Q_o = [C^T \quad A^T C^T] \qquad C^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix} \qquad C^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$Q_o = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad A^T C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\det(Q_o) = 0(0) - 1(1) = -1$

System is observable

Step 2: Determine the closed-loop C·E $(s-\mu_1)(s-\mu_2) = 0$

$[s - (-1.8 + 2.4j)][s - (-1.8 - 2.4j)] = 0 \qquad j^2 = -1$

$(s + 1.8 - 2.4j)(s + 1.8 + 2.4j) = 0$

$s^2 + 1.8s + \cancel{2.4sj} + 1.8s + 3.24 + \cancel{4.32j} - \cancel{2.4sj} - \cancel{4.32j} - 5.76j^2 = 0$

$s^2 + 3.6s + 3.24 + 5.76 = 0$

$s^2 + 3.6s + 9 = 0$

Compare with $s^2 + \alpha_1 s + \alpha_2 I = 0 \qquad \alpha_1 = 3.6 \qquad \alpha_2 = 9$

Ke = observer gain matrix

$Ke = \begin{bmatrix} Ke_1 \\ Ke_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Step 3: Determine $|sI - A + KeC| = 0$

$\left| s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} [0 \quad 1] \right| = 0$

$\left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & x_1 \\ 0 & x_2 \end{bmatrix} \right| = 0$

$\left| \begin{bmatrix} s & -20.6 + x_1 \\ -1 & s + x_2 \end{bmatrix} \right| = 0$

$$\begin{vmatrix} s & -20.6 + X_1 \\ -1 & s + X_2 \end{vmatrix} = 0$$

$$+s(s + X_2) - (-20.6 + X_1)(-1) = 0$$

$$s^2 + sX_2 - (20.6 - X_1) = 0$$

$$s^2 + sX_2 - 20.6 + X_1 = 0$$

$$s^2 + sX_2 - 20.6 + X_1 = 0$$

Recall from step 2 CLCE

$$s^2 + 3.6s + 9 = 0 \qquad \text{compare with}$$

$$(X_2)s - 20.6 + X_1 = 0$$

Coefficent of s: $3.6 = X_2$

$$X_2 = 3.6$$

Constant term : $-20.6 + X_1 = 9$

$$X_1 = 9 + 20.6$$

$$X_1 = 29.6$$

$$K_e = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 29.6 \\ 3.6 \end{bmatrix}$$

QUESTION FOUR

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & \lambda \end{bmatrix} \qquad \lambda = 2 \qquad R = 3$$

(a) $Q_c = [B \quad AB]$ $\qquad Q_o = [C^T \quad A^T C^T]$

$\cancel{AB = [}$ $A = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$C = [1 \quad 0]$

$$AB = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$$

$|Q_c| = 0(2) - 1(1) = 0 - 1 = -1$

System is controllable

Observability

$$C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad A^T = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}$$

$$A^T C^T = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$|Q_o| = 1(1) - 0(0) = 1$

System is observable

(b) $Q_c = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ $\quad Q_o = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) Reduced riccati matrix, P $\quad P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$

$$PA + A^TP + Q - PBR^{-1}B^TP = 0$$

$$PA = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & P_{11} + 2P_{12} \\ 0 & P_{21} + 2P_{22} \end{bmatrix}$$

$$A^TP = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ P_{11}+2P_{21} & P_{12} + 2P_{22} \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$PBR^{-1}B^TP = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{3} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

$$= \begin{bmatrix} P_{12} \\ P_{22} \end{bmatrix} \frac{1}{3} \begin{bmatrix} P_{21} & P_{22} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} P_{12}P_{21} & P_{12}P_{22} \\ P_{22}P_{21} & P_{22}^2 \end{bmatrix}$$

$$PA + A^TP + Q - PBR^{-1}B^TP = 0$$

$$\begin{bmatrix} 0 & P_{11}+2P_{12} \\ 0 & P_{21} + 2P_{22} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ P_{11}+2P_{21} & P_{12} + 2P_{22} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} P_{12}P_{21}/3 & P_{12}P_{22}/3 \\ P_{22}P_{21}/3 & P_{22}^2/3 \end{bmatrix} = 0$$

$1 - \frac{P_{12}P_{21}}{3} = 0 \Rightarrow 3 - P_{12}P_{21} = 0$ —— I

$P_{11} + 2P_{12} - P_{12}P_{22}/3 = 0 \Rightarrow 3P_{11} + 6P_{12} - P_{12}P_{22} = 0$ —— II

$P_{11} + 2P_{21} - P_{22}P_{21}/3 = 0 \Rightarrow 3P_{11} + 6P_{21} - P_{22}P_{21} = 0$ —— III

$P_{21} + 2P_{22} + P_{12} + 2P_{22} + 2 - \frac{P_{22}^2}{3} = 0 \Rightarrow 3P_{21} + 6P_{22} + 3P_{12} + 6P_{22} + 6 - P_{22}^2 = 0$ —— IV

$$P_{12} = P_{21}$$

Using eqn I

$3 - P_{12}P_{21} = 0$

$3 - P_{12}^2 = 0$

$3 = P_{12}^2$

$P_{12} = \sqrt{3}$

$P_{12} = P_{21} = 1.73$

Using equation II

$3P_{11} + 6P_{12} - P_{12}P_{22} = 0$

$3P_{11} + 6(1.73) - 1.73P_{22} = 0$

$3P_{11} + 10.38 - 1.73P_{22} = 0$ — V

Using equation IV

$3P_{21} + 6P_{22} + 3P_{12} + 6P_{22} + 6 - P_{22}^2 = 0$

$6P_{12} + 12P_{22} + 6 - P_{22}^2 = 0$

$6(1.73) + 12P_{22} + 6 - P_{22}^2 = 0$

$10.38 + 6 + 12P_{22} - P_{22}^2 = 0$

$-P_{22}^2 + 12P_{22} + 16.38 = 0$

$P_{22} = 13.24$ $\quad$ $P_{22} = -1.24$

neglect

To get P_{11} use eqn V

$3P_{11} + 10.38 - 1.73P_{22} = 0$

$3P_{11} = 1.73P_{22} - 10.38$

$P_{11} = \dfrac{1.73P_{22} - 10.38}{3}$

when $P_{22} = 13.24$

$P_{11} = \dfrac{1.73(13.24) - 10.38}{3} = 4.18$

$$P = \begin{bmatrix} 4.18 & 1.73 \\ 1.73 & 13.24 \end{bmatrix}$$

(d) Optimal feedback gain matrix, K

$K = R^{-1}B^TP$

$= \frac{1}{3}\begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 4.18 & 1.73 \\ 1.73 & 13.24 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1.73 & 13.24 \end{bmatrix} = \begin{bmatrix} 0.58 & 4.41 \end{bmatrix}$

(e) Closed-loop eigen value,

$|sI - A + BK| = 0$

$$\left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0.58 & 4.41 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} s & -1 \\ 0 & s-2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0.58 & 4.41 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} s & -1 \\ 0.58 & s+2.41 \end{vmatrix} = 0$$

$s - 2 + 4.41$

$s + 2.41$

$$+s(s+2.41) - (-1 \times 0.58) = 0$$

$$s^2 + 2.41s + 0.58 = 0$$

$$s = -0.27$$

$$s = -2.14$$

QUESTION FIVE

To check $-2, -1, 0, 1, 2$

QUESTION SIX

(a) positive definite $= \begin{cases} +\text{ve value}, & \text{for all values of } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$

negative definite $= \begin{cases} -\text{ve value}, & \text{for all values of } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$

i $V(x) = -x_1^2 - (3x_1 + 2x_2)^2 \longrightarrow$ −ve definite

ii $V(x) = x_1^2 + x_2^2 + \dots + x_n^2 \longrightarrow$ +ve definite

iii $V(x) \longrightarrow$ −ve definite

(iv) $V(x) \longrightarrow$ −ve definite (b.) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\qquad V(x) = x_1^2 + x_2^2$

Lyapunov first theorem

→ $V(x)$ should be +ve definite

→ $\dot{V}(x)$ should be -ve definite

$V(x) = x_1^2 + x_2^2 \longrightarrow$ +ve definite

$$\frac{\partial V(x)}{\partial x_1} = 2x_1$$

$$\dot{V}(x) = \frac{\partial V(x)}{\partial x_1} \cdot \dot{x}_1 + \frac{\partial V(x)}{\partial x_2} \cdot \dot{x}_2$$

$$\frac{\partial V(x)}{\partial x_2} = 2x_2$$

$$= 2x_1 \cdot \dot{x}_1 + 2x_2 \cdot \dot{x}_2$$

From the given matrix $\quad \dot{x}_1 = (0 \times x_1) + (1 \times x_2) = 0 + x_2 = x_2$

$\dot{x}_2 = (-1 \times x_1) + (1 \times x_2) = -x_1 + x_2$

$$\dot{V}(x) = 2x_1 \cdot x_2 + 2x_2(-x_1 + x_2)$$

$$= 2x_1x_2 - 2x_1x_2 + 2x_2^2 \longrightarrow \text{+ve definite}$$

The system is unstable

(c) $A = \begin{bmatrix} -1 & -2 \\ 1 & -4 \end{bmatrix} \quad Q = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$

$$PA + A^T P = -Q$$

$$\cancel{\begin{bmatrix} -1 & -2 \\ 1 & -4 \end{bmatrix}}\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & -4 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ -2 & -4 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = -\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$\underline{\text{NOTE}}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -Q$

Refer to my note for complete solution

Solved by IB
Captured by D2G.